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1A

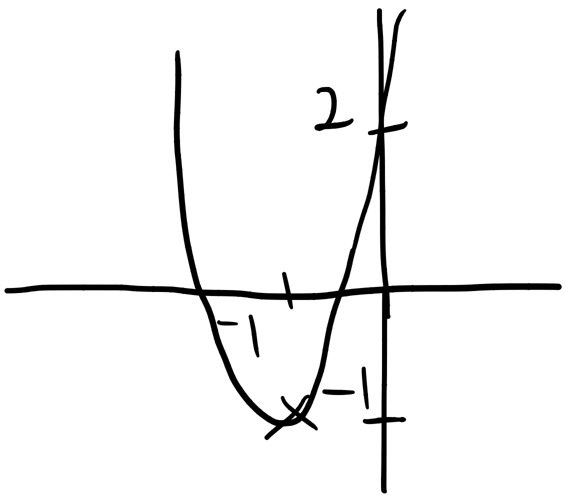
1. b) $y = 3x^2 + 6x + 2$

$= 3\left(x^2 + 2x + \frac{2}{3}\right)$

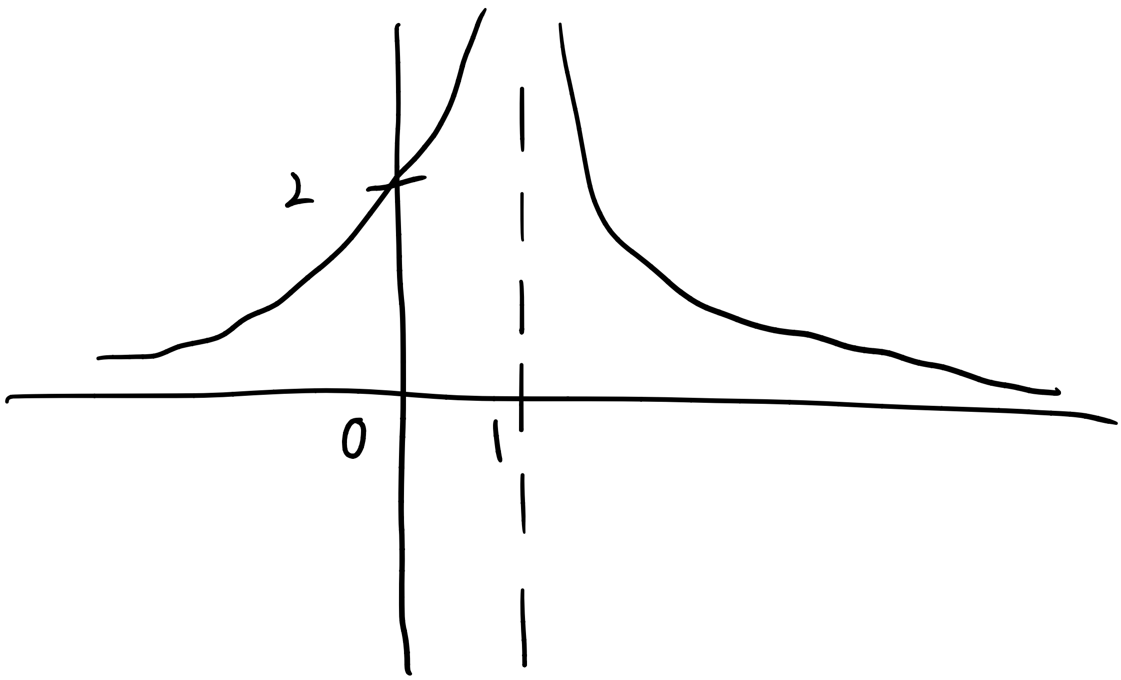
$= 3\left(x^2 + 2x + 1 - 1 + \frac{2}{3}\right)$

$= 3\left((x+1)^2 - \frac{1}{3}\right)$

$= 3(x+1)^2 - 1$



2. b) $y = \dfrac{2}{(x-1)^2}$



3. a) $\dfrac{x^3+3x}{1-x^4}$ $\quad f(-x) = \dfrac{-x^3-3x}{1-x^4}$

$= -f(x)$

$\therefore$ odd

b) $\sin^2 x$ $\quad f(-x) = \sin^2 x$

$\therefore$ even

e) $J_0(x^2)$ $\quad f(-x) = J_0(x^2)$

$\therefore$ even

6. b) $\sin x - \cos x$ $\quad \sin(a+b)$

$= \sin a \cos b + \cos a \sin b$

$\cos y = 1 \quad \sin y = -1$

$\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)$

$= \sqrt{2}\left(\sin\left(x + \frac{7\pi}{4}\right)\right)$

$= \sqrt{2}\sin\left(x + \frac{7\pi}{4}\right)$

$\sin\frac{7\pi}{4}$

$= -\sin\left(2\pi - \frac{7\pi}{4}\right)$

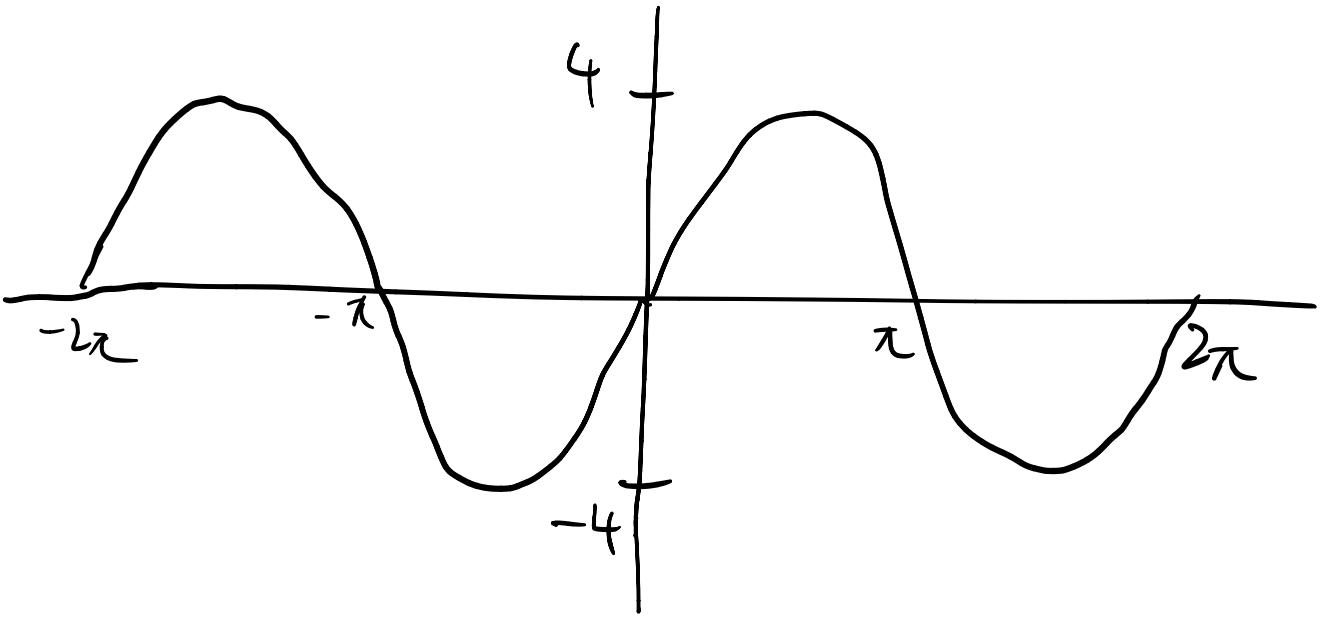
$= -\sin\left(\frac{\pi}{4}\right)$

7. b) $-4\cos\left(x+\frac{\pi}{2}\right)$

Period $= 2\pi$

Amplitude $= 4$

Phase angle $= -\frac{\pi}{2}$



1B

1. $d = 16t^2$

a) $d(2) = 16(2)^2$

$= 64$

$\bar{v} = \dfrac{64-0}{2-0} = 32 \text{ ft/s}$

b) $16t^2 = 400$

$16(t^2 - 25) = 0$

$t^2 = 25$

$\therefore t = \pm 5$

$= 5$

$\bar{v} = \dfrac{16(5)^2 - 16(3)^2}{5-3} = \dfrac{400-144}{2} = 128 \text{ ft/s}$

c) $\dfrac{dv}{dt} = 32t \quad 32(5) = 160 \text{ ft/s}$

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1C

1. a) $A = \pi r^2$

$$\lim_{\Delta r \to 0} \frac{f(r+\Delta r) - f(r)}{\Delta r}$$

$$= \frac{\pi (r+\Delta r)^2 - \pi r^2}{\Delta r}$$

$$= \frac{\pi r^2 + 2\pi r \Delta r + \pi \Delta r^2 - \pi r^2}{\Delta r}$$

$$= \lim_{\Delta r \to 0} 2\pi r + \pi \Delta r$$

$$= 2\pi r$$

3. a) $f(x) = \dfrac{1}{2x+1}$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{\dfrac{1}{2(x+\Delta x)+1} - \dfrac{1}{2x+1}}{\Delta x}$$

$$= \frac{1}{\Delta x} \cdot \frac{2x+1-2x-2\Delta x-1}{(2x+2\Delta x+1)(2x+1)}$$

$$= \frac{1}{\cancel{\Delta x}} \cdot \frac{-2\cancel{\Delta x}}{4x^2+2x+4\Delta x \cdot x+2\Delta x+2x+1}$$

$$= \lim_{\Delta x \to 0} -\frac{2}{4x^2+4x+4\Delta x \cdot x+2\Delta x+1}$$

$$= -\frac{2}{4x^2+4x+1} \qquad f'(x) = -\frac{2}{(2x+1)^2}$$

b) $f(x) = 2x^2 + 5x + 4$

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \frac{2(x+\Delta x)^2 + 5(x+\Delta x) + 4 - 2x^2 - 5x - 4}{\Delta x}$$

$$= \frac{\cancel{2x^2} + 4x\cdot\Delta x + 2\Delta x^2 \cancel{+5x} + 5\Delta x \cancel{+4} \cancel{-2x^2} \cancel{-5x} \cancel{-4}}{\Delta x}$$

$$= \frac{\Delta x(4x + 2\Delta x + 5)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 4x + 2\Delta x + 5$$

$$= 4x + 5$$

e) a $f'(x) = -\frac{2}{(2x+1)^2}$

$f'(x) = 1$ $f'(x) = -1$ $f'(x) = 0$

$-\frac{2}{(2x+1)^2} = 1$ $\frac{2}{(2x+1)^2} = 1$ $-\frac{2}{(2x+1)^2} = 0$

$(2x+1)^2 = -2$

$2x+1 = \pm\sqrt{2}$

$2x = -1 \pm \sqrt{2}$

$x = \frac{-1 \pm \sqrt{2}}{2}$

b $f'(x) = 4x+5$

$f'(x) = -1$ $f'(x) = 1$ $f'(x) = 0$

$4x+5 = -1$ $4x+5 = 1$ $4x+5 = 0$

$4x = -6$ $4x = -4$ $4x = -5$

$x = -\frac{3}{2}$ $x = -1$ $x = -\frac{5}{4}$

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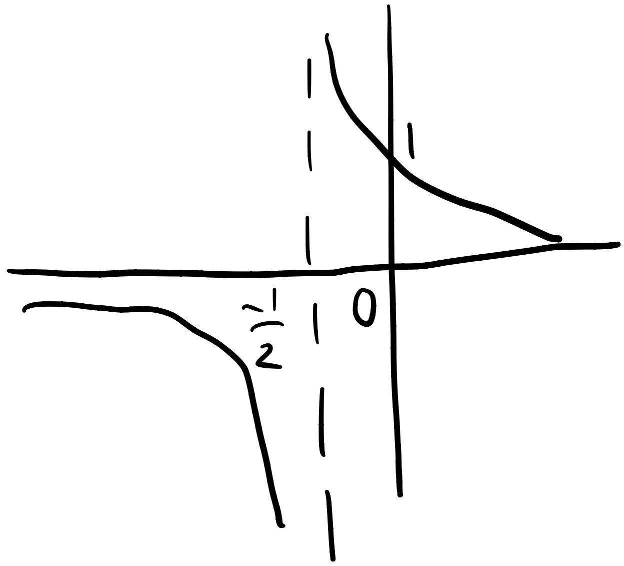
4.

a) $f(x) = \dfrac{1}{2x+1}$, $x = 1$

$f(x) = (2x+1)^{-1}$



$f'(x) = -(2x+1)^{-2} \cdot 2$

$= -\dfrac{2}{(2x+1)^2}$

When $x = 1$, $f'(x) = -\dfrac{2}{9}$, $f(x) = \dfrac{1}{3}$

$y = mx + c$

$\dfrac{1}{3} = -\dfrac{2}{9}(1) + c$

$c = \dfrac{3}{9} + \dfrac{2}{9} = \dfrac{5}{9}$

$\therefore y = -\dfrac{2}{9}x + \dfrac{5}{9}$

b) $f(x) = 2x^2 + 5x + 4$, $x = a$

$f'(x) = 4x + 5$

When $x = a$, $f(a) = 2a^2 + 5a + 4$, $f'(a) = 4a + 5$

$y - y_1 = m(x - x_1)$

$y - 2a^2 - 5a - 4 = m(x - a)$

$y = (4a+5)(x-a) + 2a^2 + 5a + 4$

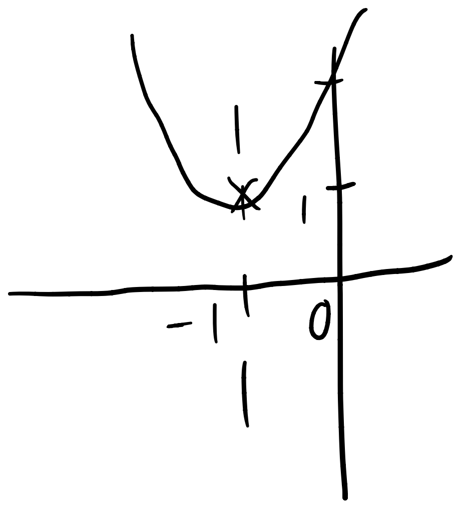
$= 4ax + 5x - 4a^2 \cancel{- 5a} + 2a^2 \cancel{+ 5a} + 4$

$= (4a+5)x - 2a^2 + 4$

5.

$$y = 1 + (x-1)^2$$
$$= 1 + x^2 - 2x + 1$$
$$= x^2 - 2x + 2$$

$$\frac{dy}{dx} = 2x - 2$$



$$y - y_1 = m(x - x_1)$$

$$y - 0 = m(x - 0)$$

$$y = mx$$

$$y = (2x-2)x$$

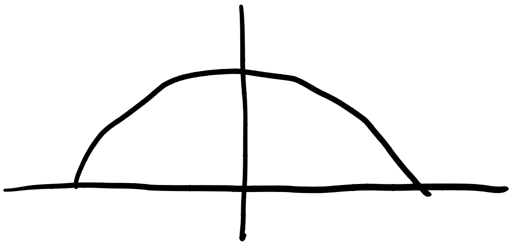
$$y = 2x^2 - 2x$$

$$y = mx + c$$

$$y = (2x-2)x$$
$$= 2x^2 - 2x$$

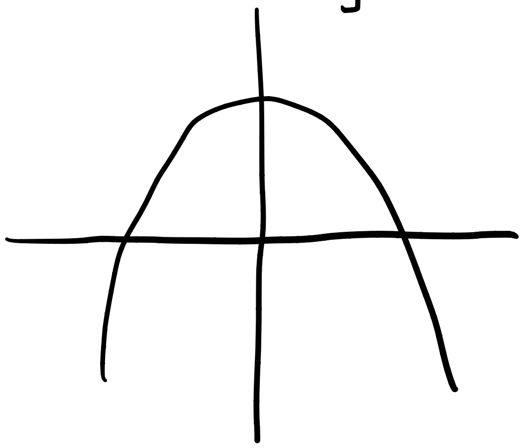
6.

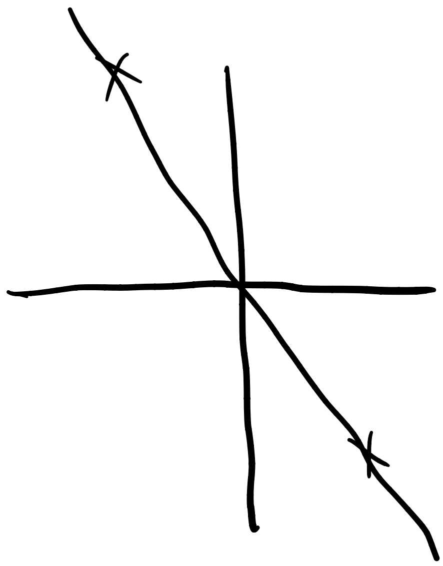
a)



$y=-x^2-c$

b)





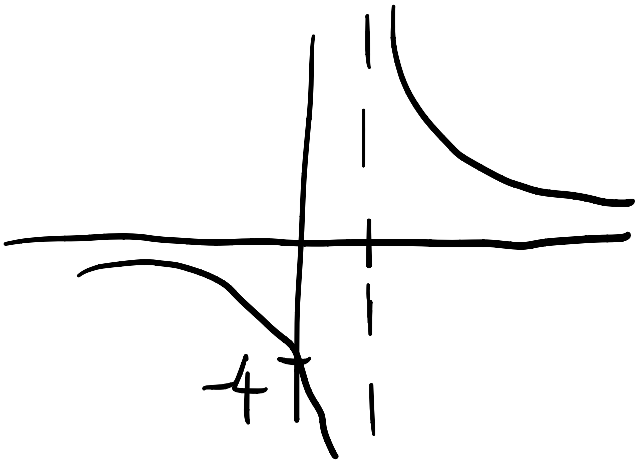
1D

16/1/2025

1a) $\lim_{x \to 0} \frac{4}{x-1}$

$= \frac{4}{-1}$

$= -4$



c) $\lim_{x \to -2} \frac{4x^2}{x+2}$

$$\begin{array}{r} 4x - 8 \\ x+2 \,\big)\, 4x^2 + 0 \\ 4x^2 + 8x \\ \hline -8x + 0 \\ -8x - 16 \\ \hline 16 \end{array}$$

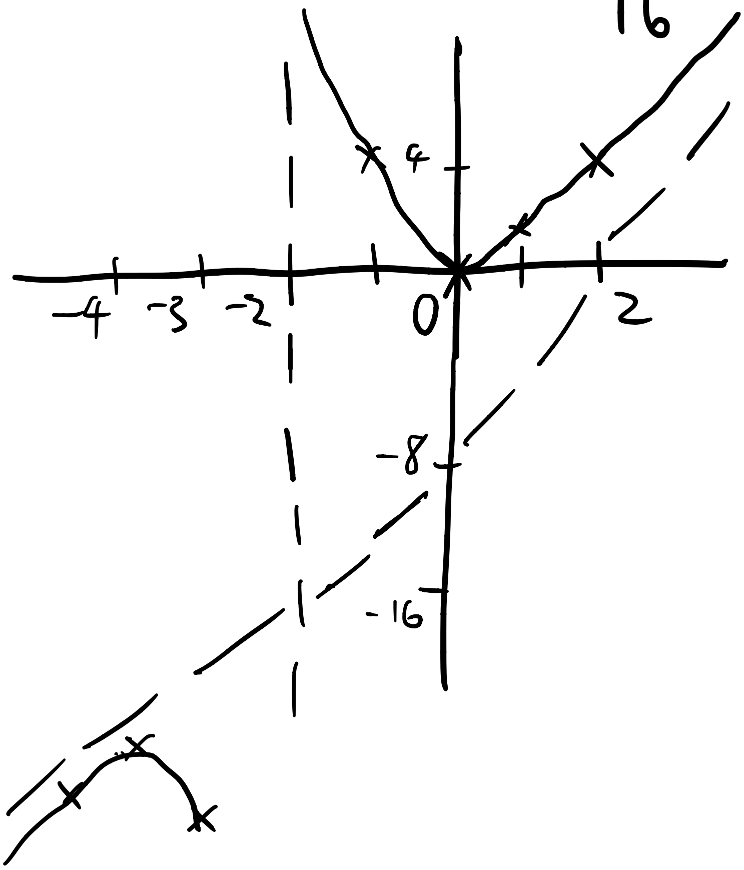
$= \lim_{x \to -2} 4x - 8 + \frac{16}{x+2}$

-3	-20	$-16 = -36$
-4	-24	$-8 = -32$
-5	-28	$-\frac{16}{3} = -33\frac{1}{3}$

limit does not exist

$\Rightarrow \lim_{x \to -2^+} = \infty$

$\lim_{x \to -2^-} = -\infty$



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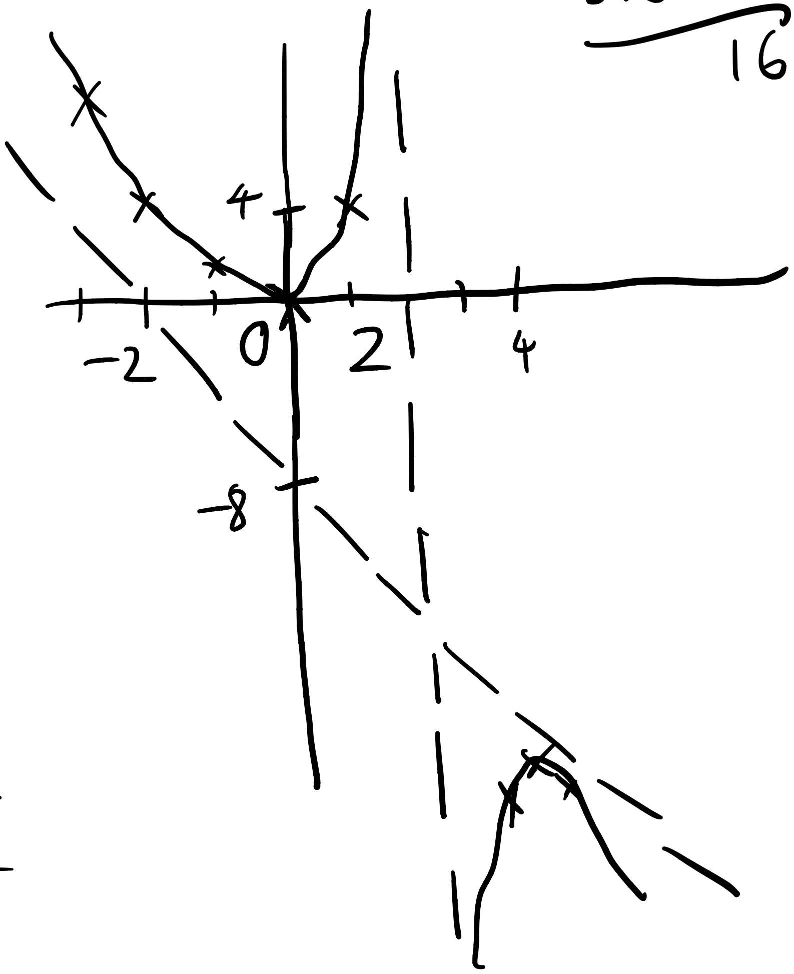
d) $\lim\limits_{x \to 2^+} \dfrac{4x^2}{2-x}$

$= \lim\limits_{x \to 2^+} - \dfrac{4x^2}{x-2}$

$$\begin{array}{r} 4x + 8 \\ x-2 \overline{) 4x^2 + 0 + 0} \\ 4x^2 - 8x \\ \hline 8x + 0 \\ 8x - 16 \\ \hline 16 \end{array}$$

$= \lim\limits_{x \to 2^+} -4x - 8 - \dfrac{16}{x-2}$

x	
1	$-12 - \dfrac{16}{-1} = 4$
3	$-20 - 16 = -36$
-1	$-4 - \dfrac{16}{-3}$
	$= -4 + 5\frac{1}{3} = \frac{4}{3}$
-2	4
-3	$4 + \dfrac{16}{5} = 7\frac{1}{5}$
4	$-24 - 8 = -32$
5	$-28 - \dfrac{16}{3} = -33\frac{1}{3}$



$\Rightarrow$ limit does not exist

$\lim\limits_{x \to 2^+} \dfrac{4x^2}{2-x}$

$= -\infty$

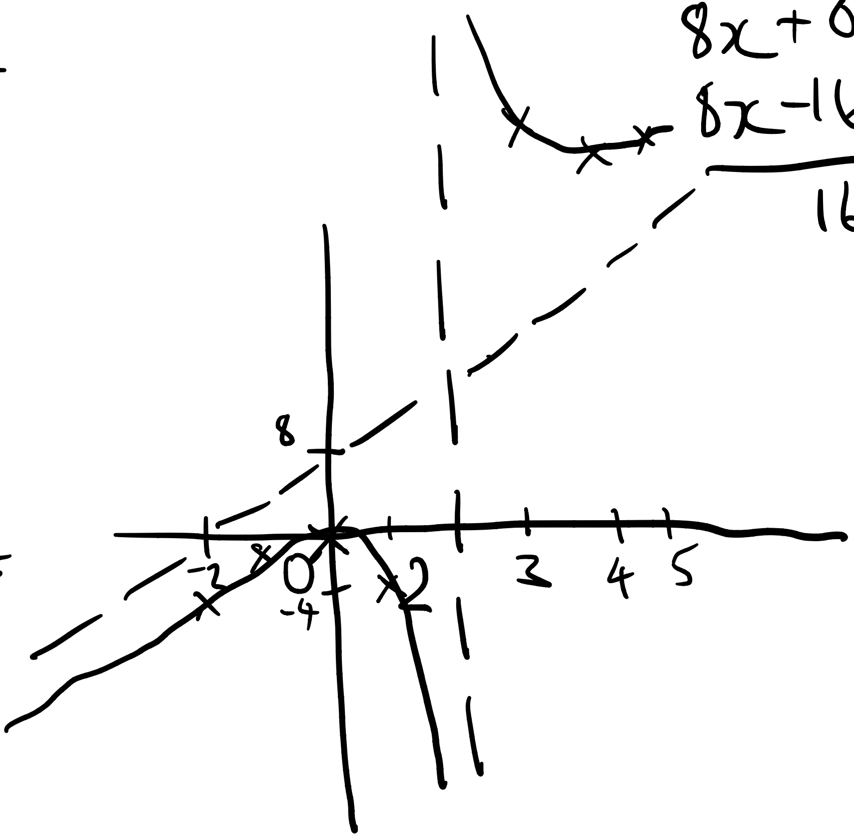
f) $\lim\limits_{x\to\infty} \dfrac{4x^2}{x-2}$

$= -\lim\limits_{x\to\infty} \dfrac{4x^2}{-x+2}$

$$
\begin{array}{r}
4x + 8 \\
x-2 \,\big)\, 4x^2 + 0 + 0 \\
4x^2 - 8x \\
\hline
8x + 0 \\
8x - 16 \\
\hline
16
\end{array}
$$

$= \lim\limits_{x\to\infty} 4x+8 + \dfrac{16}{x-2}$

x	y
1	-4
-1	$-1\frac{1}{3}$
3	36
4	32
5	$28 + 5\frac{1}{3} = 33\frac{1}{3}$



$\Rightarrow \lim\limits_{x\to\infty} \dfrac{4x^2}{x-2}$

Approaches + values of $4x+8$

$= \infty$

g) $\lim_{x \to \infty} \frac{4x^2}{x-2} - 4x$

$= \lim_{x \to \infty} 4x + 8 + \frac{16}{x-2} - 4x$

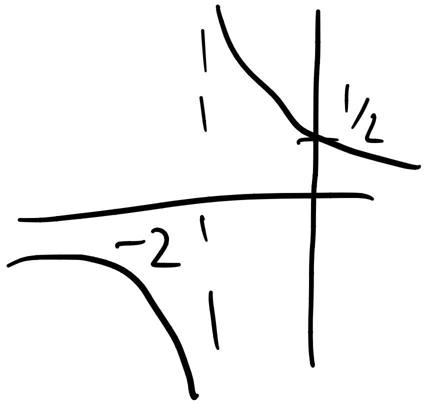
$= \lim_{x \to \infty} \frac{16}{x-2} + 8$

$= 8$

5/2/25

3 a) $\frac{x-2}{x^2-4} = \frac{x-2}{(x+2)(x-2)}$

$= \frac{1}{x+2}$



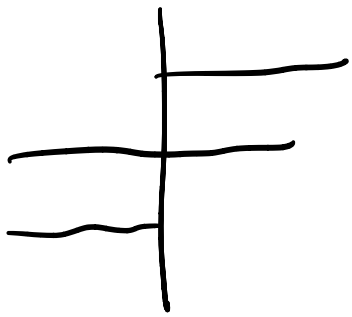
$\therefore$ Infinite discontinuity

c) $\frac{x^4}{\cancel{x^3}} = x$ $\quad\therefore$ continuous

d) $f(x) = \begin{cases} x + a, & x > 0 \\ -x + a, & x < 0 \end{cases}$

$\therefore$ Continuous

e) $f'(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$



$\therefore$ Jump Discontinuity

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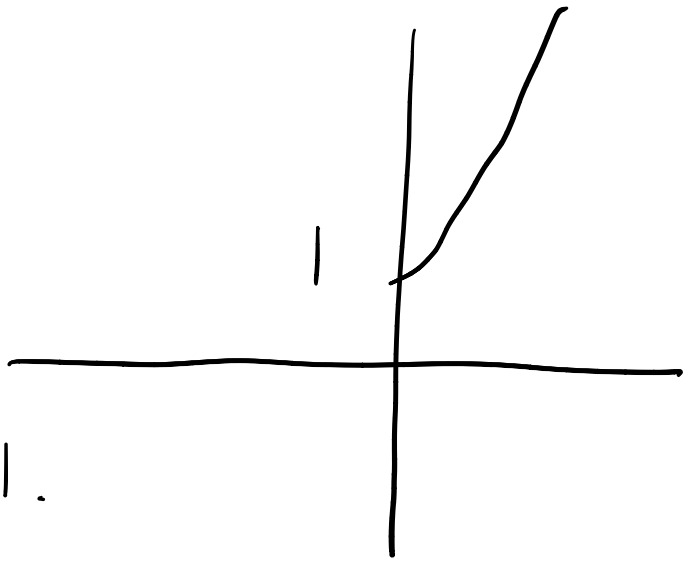
6. a) $f(x) = \begin{cases} x^2+4x+1, & x \geq 0 \\ ax+b, & x < 0 \end{cases}$



When $x=0$, $y=1$.

$\Rightarrow a(0)+b=1$

$\therefore b=1$

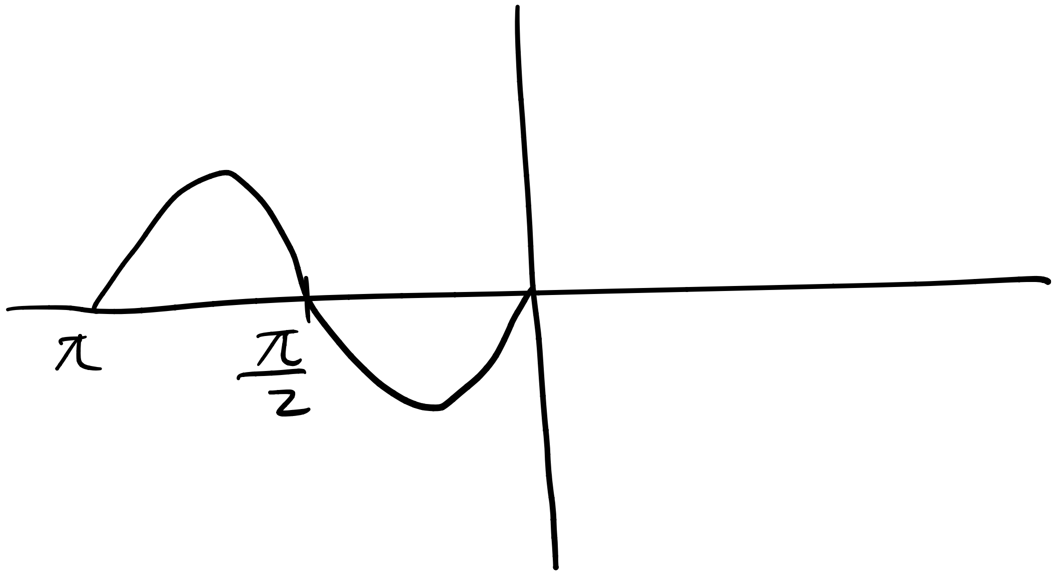
$f_1'(x) = a$
$x \to 0$

$f_2'(x) = 2x+4$
$x \to 0$

$a=4$

$\therefore 4x+1$

8. a) $f(x) = \begin{cases} ax+b, & x>0 \\ \sin 2x, & x \le 0 \end{cases}$



$f(0) = \sin 2(0)$
$= 0$

$f(0) = a(0)+b$
$= b$

$\therefore b = 0$

$\lim\limits_{x \to 0^-} 2\cos 2x = \lim\limits_{x \to 0^+} a$

$2 = a$

If $f(x)$ is differentiable, $a = 2$.

$\therefore b = 0, \quad a = \{x \in R, x \neq 2\}$

1E

25/2/25

1. a)

$$\frac{dy}{dx} = 10x^9 + 15x^4 + 6x^2$$

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c) $$\frac{dy}{dx} = \frac{1}{2}$$

2. b) $$\frac{dy}{dx} = x^6 + 5x^5 + 4x^3$$

$$y = \frac{1}{7}x^7 + \frac{5}{6}x^6 + x^4$$

3. $$y = x^3 + x^2 - x + 2$$

$$\frac{dy}{dx} = 3x^2 + 2x - 1 = 0$$

$$(3x-1)(x+1) = 0$$

$$\therefore (-1, 3), \left(\frac{1}{3}, \frac{49}{27}\right)$$

4.

b) $f(x) = \begin{cases} ax^2+bx+4, & x \leq 1 \\ 5x^5+3x^4+7x^2+8x+4, & x > 1 \end{cases}$

$$\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^+} f'(x)$$

$$\lim_{x\to 1^-} 2ax+b = \lim_{x\to 1^+} 25x^4+12x^3+14x+8$$

$$2a+b = 25+12+14+8$$

$$2a = 59-b$$

$$a = \frac{59-b}{2}$$

$$f(1) = \lim_{x\to 1} 5x^5+3x^4+7x^2+8x+4$$

$$a+b+4 = 5+3+7+8+4$$

$$a+b = 23$$

$$\frac{59-b}{2} = 23-b$$

$$59-b = 46-2b$$

$$b = -13$$

$\therefore a = 36,\ b = -13$

1J

16/1/2025

1e) $y = \frac{\sin x}{x}$

$$\frac{dy}{dx} = \frac{\cos x (x) - \sin x (1)}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

2) $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}}$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\cos x - \cos\frac{\pi}{2}}{x - \frac{\pi}{2}}$$

$$= \lim_{\Delta x \to 0} \frac{\cos\left(\frac{\pi}{2} + \Delta x\right) - \cos\frac{\pi}{2}}{\left(\frac{\pi}{2} + \Delta x\right) - \frac{\pi}{2}}$$

$$= \frac{d}{dx}(\cos x)\Big|_{x=\frac{\pi}{2}}$$

$$= -\sin x\Big|_{x=\frac{\pi}{2}}$$

$$= -\sin\frac{\pi}{2}$$

$$= -1$$

1F

7/2/25

1. a) $y = (x^2+2)^2$, $u = x^2+2$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 2(x^2+2) \cdot (2x)$$

$$= 4x^3 + 8x$$

$$\frac{dy}{dx} = \frac{f(x+\Delta x) + f(x)}{\Delta x}$$

$$\lim_{\Delta x \to 0} f(x) = \frac{((x+\Delta x)^2+2)^2 + (x^2+2)^2}{\Delta x}$$

$$= \frac{(x+\Delta x)^4 + 4(x+\Delta x)^2 + 4 + x^4 + 4x^2 + 4}{\Delta x}$$

$$= \frac{x^4 + 4x^3\Delta x + 6x^2\Delta x^2 + 4x \cdot \Delta x^3 + \Delta x^4 + 4x^2 + 8x\Delta x + 4\Delta x^2 + 4 + x^4 + 4x^2 + 4}{\Delta x}$$

$$= \frac{2x^4 + 4x^3\Delta x + 6x^2\Delta x^2 + 4x\Delta x^3 + \Delta x^4 + 8x^2 + 8x\Delta x + 4\Delta x^2 + 8}{\Delta x}$$

(?)

b) $(x^2+2)^{100}$

$$\frac{dy}{dx} = 100(x^2+2)^{99} \cdot 2x$$

$$= 200x(x^2+2)^{99}$$

$f(x) = g(x) \cdot h(x)$

$f'(x) = g'(x)h(x) + h'(x)g(x)$

2. $x^{10} \cdot (x^2+1)^{10}$

$$\frac{dy}{dx} = 10x^9(x^2+1)^{10} + 10(x^2+1)^9 2x \cdot x^{10}$$

$$= 10(x^2+1)^9(x^9(x^2+1) + 2x^{11})$$

$$= 10x^9(x^2+1)^9((x^2+1) + 2x^2)$$

$$= 10x^9(x^2+1)^9(3x^2+1)$$

6.

If $f(x)$ is even,

$\Rightarrow f(x) = f(-x)$

$\Rightarrow f'(x) = f'(-x)$ $\quad f'(-x) = -f'(-x)$

$f'(-x) = -f'(-x)$

$\Rightarrow f'$ is odd

If $g(x)$ is odd,

$\Rightarrow g(x) = -g(-x)$

$g'(x) = -g'(-x)$ $\quad -g'(-x) = -1 \cdot -g'(-x)$

$= g'(-x)$

$\Rightarrow g'(x) = g'(-x)$

$\Rightarrow g'$ is even.

7.

b) $$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{dm}{dv} = \frac{0 - \frac{1}{\cancel{2}}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \cdot \left(-\frac{\cancel{2}}{c^2}v\right) m_0}{1 - \frac{v^2}{c^2}}$$

$$= \frac{\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \cdot \frac{v m_0}{c^2}}{1 - \frac{v^2}{c^2}}$$

$$= \frac{\frac{v m_0}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}\left(1 - \frac{v^2}{c^2}\right)}$$

$$= \frac{v m_0}{c^2 \left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

c) $F = \dfrac{mg}{(1+r^2)^{\frac{3}{2}}}$

$$\frac{dF}{dr} = \frac{0 - \frac{3}{2}(1+r^2)^{\frac{1}{2}} \cdot 2r \cdot mg}{(1+r^2)^3}$$

$$= \frac{-3rmg\,(1+r^2)^{\frac{1}{2}}}{(1+r^2)^3}$$

$$= \frac{-3rmg}{(1+r^2)^{\frac{5}{2}}}$$

10/2/25

1J

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec x$

$1 + \cot^2 x = \csc x$

1. a) $\sin(5x^2)$

$$\frac{dy}{dx} = 10x \cdot \cos(5x^2)$$

k) $\tan^2(3x)$

$$y = \frac{\sin^2(3x)}{\cos^2(3x)}$$

$$= \frac{2\sin(3x) \cdot 3\cos(3x) \cdot \cos^2(3x) - 2\cos(3x) \cdot 3(-\sin 3x)}{\cos^4(3x)}$$

$$= \frac{6\sin(3x)\cos(3x)\left(\cos^2(3x) + 1\right)}{\cos^4(3x)}$$

$$= \frac{6\sin(3x)\left(\cos^2(3x) + 1\right)}{\cos^3(3x)}$$

×

m)

$\cos(2x)$

$\cos^2 x - \sin^2 x$

$\frac{dy}{dx} = -2\sin 2x$

$\frac{dy}{dx} = 2\cos x(-\sin x) - 2\sin x \cdot \cos x$

$= -4\sin x\cos x$

$= -2\sin 2x$

$2\cos^2 x$

$\frac{dy}{dx} = 4\cos x(-\sin x)$

$= -4\sin x\cos x$

$= -2\sin 2x$

$\therefore$ No, because it is only their derivatives that are the same.